

Circles and Ratio Summary

KEY TERMS

- congruent
- circle
- radius
- diameter
- circumference
- pi
- unit rate

LESSON

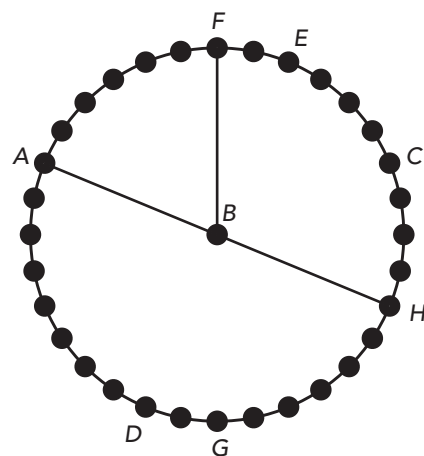
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Pi: The Ultimate Ratio

A **circle** is a collection of points on the same plane equidistant from the same point. The center of a circle is the point from which all points on the circle are equidistant.

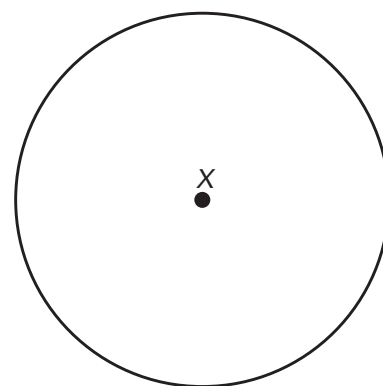
A **radius** of a circle is a line segment formed by connecting a point on the circle and the center of the circle. The distance across a circle through the center is a **diameter** of the circle. A diameter of a circle is a line segment formed by connecting two points on the circle such that the line segment passes through the center point.

Circles are named by their center point. For example, the circle shown is Circle *B*. A radius of Circle *B* is line segment *FB*. A diameter of Circle *B* is line segment *AH*.



The distance around a circle is called the **circumference** of the circle. The number **pi** (π) is the ratio of the circumference of a circle to its diameter. That is, $\pi = \frac{\text{circumference of a circle}}{\text{diameter of a circle}}$, or $\pi = \frac{C}{d}$, where *C* is the circumference of the circle, and *d* is the diameter of the circle. The number π has an infinite number of decimal digits that never repeat. Some approximations used for the value π are 3.14 and $\frac{22}{7}$. You can use the ratio to write a formula for the circumference of a circle: $C = \pi d$.

Congruent means that it has the same shape and size. For example, Circle X is congruent to Circle B . If line segment AH on Circle B has a length of 10 centimeters, then the circumference of Circle X is $C = \pi(10)$ centimeters, or approximately 31.4 centimeters.



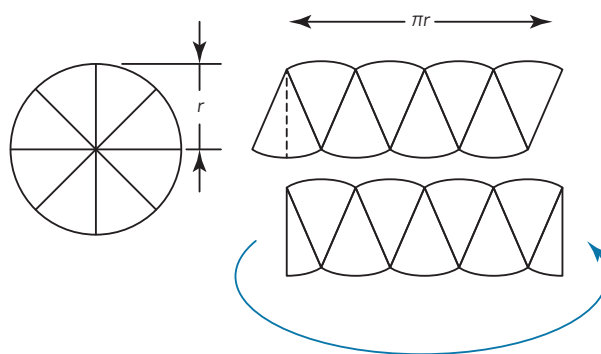
LESSON

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That's a Spicy Pizza!

The circumference of a circle is the distance around the circle, while the area of a circle is the amount of space contained inside the circle. The formula for the area of a circle is $A = \pi r^2$.

The area formula for a circle can be derived by dividing a circle into a large number of equal-sized wedges. Laying these wedges as shown, you can see that they will form an approximate rectangle with a length of πr and a height of r .



A **unit rate** is a ratio of two different measures in which either the numerator or denominator is 1.

For example, a large pizza with a diameter of 18 inches costs \$14.99. The rate of area to cost is $\frac{\pi \cdot 9^2}{14.99} = \frac{81\pi}{14.99}$. Using 3.14 for π , the unit rate is approximately 16.97 square inches per dollar. The unit rate of cost to area is $\frac{1}{16.97}$, or approximately \$0.06 per square inch.

Circular Reasoning

Given a specific length to form a perimeter or circumference, arranging that length into the shape of a circle provides the maximum area.

For example, suppose you have 176 feet of fencing to use to fence off a portion of your backyard for planting vegetables. You want to maximize the amount of fenced land. Calculate the maximum fenced area you will have.

The length of fencing you have will form the circumference of a circle.

$$C = \pi d$$

Use the formula for the circumference of a circle to determine the diameter of the fenced area.

$$176 = \pi d$$

$$56 \approx d$$

If the diameter of the fenced area is about 56 feet, the radius is 28 feet. Use this information to calculate the area of the fenced land.

$$A = \pi r^2$$

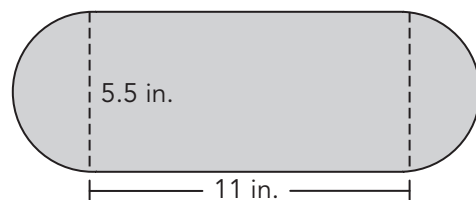
$$A = \pi \cdot 28^2$$

The maximum fenced area you will have is about 2461.76 square feet.

$$A = 784\pi \approx 2461.76$$

Many geometric figures are composed of two or more geometric shapes. These figures are known as composite figures. When solving problems involving composite figures, it is often necessary to calculate the area of each figure and then add these areas together.

For example, a figure is composed of a rectangle and two semi-circles. Determine the area of the figure.



Calculate the area of the rectangle.

$$A = l \times w$$

$$A = (11)(5.5)$$

$$A = 60.5 \text{ square inches}$$

The two semi-circles together make one circle.

Calculate the area of the circle.

$$A = \pi r^2$$

$$A = \pi(2.75)^2$$

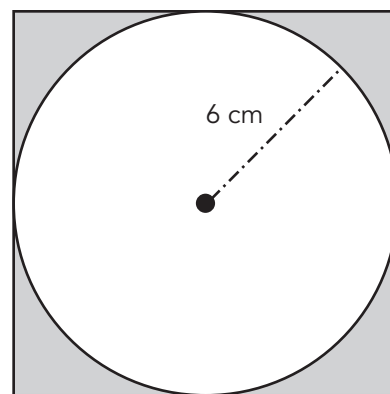
$$A = 7.5625\pi \approx 23.75 \text{ square inches}$$

The area of the composite figure is approximately 60.5 square inches plus 23.75 square inches, or 84.55 square inches.

When determining the area of a shaded region of a figure, it is often necessary to calculate the area of a figure and subtract it from the area of a second figure.

For example, this figure shows a circle inscribed in a square. Determine the area of the shaded region.

When a circle is inscribed in a square, the diameter of the circle is equal to the side length of the square.



Calculate the area of the square.

$$A = s^2$$

$$A = 12^2$$

$$A = 144 \text{ square centimeters}$$

Calculate the area of the circle.

$$A = \pi r^2$$

$$A = \pi(6)^2$$

$$A = 36\pi \approx 113.04 \text{ square inches}$$

The area of the shaded region is approximately 144 square centimeters minus 113.04 square centimeters, or 30.96 square centimeters.